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Temperature states, ground states and relativistic vacuum states in the context of symmetry breakdown

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Abstract. We discuss some of the similarities (respectively differences) between temperature states, ground states and relativistic vacuum states in the context of spontaneous symmetry breakdown.

Temperature ($\beta < \infty$) states (or KMS states) in statistical mechanics display remarkable properties, sometimes qualitatively different from ground ($\beta = \infty$) states and the vacuum in relativistic quantum field theories (RQFT). Some of these differences appear in the framework of scattering theory of quasi-particles (Narnhofer *et al* 1983). There are also some similarities, apparent, for instance, in the recent versions of Goldstone's theorem for $\beta < \infty$ (Landau *et al* 1981, Martin 1982) and $\beta = \infty$ (Landau *et al* 1981), the latter being the non-relativistic analogue of a well known result in RQFT (Ezawa and Swieca 1967). It is our purpose in this paper to analyse some of these similarities (resp. differences) more systematically, in the natural context of spontaneous symmetry breaking. In the process, we formulate and prove some ($\beta < \infty$ and $\beta = \infty$) statistical mechanical counterparts to certain results in RQFT, such as Coleman's theorem and properties of the charge operator (see, e.g. Swieca 1970 and references given there), which have not been considered elsewhere. Although not all of them are physically relevant, we hope that our discussion clarifies some of the interesting structural differences between these states. This may also be of some interest because models in statistical mechanics are often used as guides to some aspects of the behaviour of RQFT.

As to the notion of symmetry, a group of strongly continuous automorphisms γ_g of the algebra of quasi local observables $\mathscr{A}$ is assumed to exist|| with the additional property that it commutes with the time evolution α_t, i.e.

$$\gamma_g \cdot \alpha_t(A) = \alpha_t \cdot \gamma_g(A), \qquad A \in \mathscr{A}.$$

Already this algebraic concept involves a couple of hidden physical assumptions, in particular with respect to the range of interaction (Requardt 1982).

The symmetry is called spontaneously broken when the state representing the vacuum (resp. the ground or temperature state) is not left invariant by the symmetry. This implies automatically that γ_g cannot be implemented by a group of unitary

§ Supported in part by CNPq.

|| Good papers to refer to for the application of C^*- and von Neumann algebras to statistical mechanics are Hugenholtz (1972) and Bratelli and Robinson (1981).

operators in the representation space, in particular suitable infinitesimal generators of the unitary group do not exist. To see this we have to remember how the global generators of the symmetry are usually defined.

In the case of a continuous symmetry we are usually given in each concrete representation a *formal* infinitesimal generator expressed as an integral $\int q(x, t)\,\mathrm{d}^3x$ over a certain operator density $q(x, t)$ where q can frequently be inferred from the physical context (e.g. charge density, energy-momentum density etc). In RQFT $q(x, t)$ is a local operator-valued distribution, in many-body physics it may have non-local features as for example the energy density (incorporating the usually non-local interaction potential).

To give the global generator Q a rigorous meaning one has to approximate it by a certain localised sequence

$$Q_R(t) := \int q(x, t) f_R(x)\,\mathrm{d}^3x, \qquad R \to \infty$$

with a suitably defined sequence of functions f_R. Frequently one defines f_R by: $f_R(x) := f(|x|/R)$, f smooth with $f(s) = 1$ for $|s| < 1$, $f(s) = 0$ for $|s| > 2$ (cf e.g. Kastler *et al* 1966).

Remark. We do not discuss whether and in what sense Q, Q_R are related to e.g. $\pi(\mathscr{A})''$ and the special features arising in the context of KMS-states (there are some scattered remarks in the literature and some unpublished material by Requardt).

One now defines Q via

$$Q \cdot \pi(A)\Omega := \lim_{R\to\infty} [Q_R, \pi(A)]\Omega.$$

The above notion of symmetry guarantees that this definition is in fact time independent!

If the symmetry is *conserved* (namely $\pi(\gamma_g(A)) = U_g \cdot \pi(A) \cdot U_g^{-1}$, U_g unitary, $U_g\Omega = \Omega$) we have also $(\Omega|[Q, A]\Omega) = 0$ (since $Q\Omega = 0$). Spontaneous symmetry breaking (SSB) however implies that there exists an $A \in \mathscr{A}$ (then being dubbed a *symmetry breaking* observable) such that

$$(\Omega|\pi(\gamma_g(A))\Omega) \neq (\Omega|\pi(A)\Omega) \qquad \Rightarrow \qquad \lim_{R\to\infty} (\Omega|[Q_R, \pi(A)]\Omega) \neq 0$$

(cf the above references), which furthermore turns out to be in fact just the *characterising* property!

Remark. One should however note that within a *purely algebraic* setting the connection between e.g. Q_R, γ_g is quite subtle and one would need the rather elaborate machinery of derivations on C^* algebras. Instead of doing this we adopt the physicists' point of view and take the existence of the suitable generator densities in the various representations for granted.

As to the Goldstone theorem proper, there are two versions, one valid for $T > 0$ (Landau *et al* 1981, Martin 1982, Requardt 1980), the other for $T = 0$ (Landau *et al* 1981). In the former SSB implies a space clustering not faster than $|x|^{-\alpha}$, with $\alpha = \nu - 2$ (Martin 1982) and a characteristic singularity at zero in the *joint* energy-momentum spectrum (cf Requardt 1980). In the latter the consequence is the absence of an energy gap (Landau *et al* 1981).

The statement which for $\beta < \infty$ would seem to be in closest formal analogy (although without much physical content) to the assertion for $\beta = \infty$, namely, absence of a gap in the spectrum of the generator H_β of time-translations in the GNS representation is, however, seen to be empty because H_β is not expected to have a gap under rather general assumptions on the rate of time-like clustering (Haag *et al* 1974, proposition 3). These are seen to hold for the free Bose gas (Haag and Trych-Pohlmeyer 1977), which does exhibit SSB, as well as for the free Fermi gas (Haag *et al* 1974, which does not.

It is well known (Sirugue and Testard 1971) that the weak-star limit ω, as $\beta \to \infty$, of a net $\{\omega_\beta\}$ of KMS states satisfies the KMS condition for the ground state, $H_\omega \geq 0$. Some mathematical properties of ω are however distinct from those of the net $\{\omega_\beta\}$, and this fact has a number of implications in the theory of symmetry breakdown in statistical mechanics. We assume that we are dealing with an equilibrium state ω_β at $\beta < \infty$ (KMS state) or a ground state (at $\beta = \infty)\omega_\infty$. The C^* algebra of observables is $\mathscr{A}$, and $\Pi_\omega(\mathscr{A})$ denotes the GNS representation of $\mathscr{A}$ associated to the state ω on the Hilbert space $\mathscr{H}_\omega$, with time-translation automorphism group α_t^β (resp. α_t^∞), and generators H_β (resp. $H_\infty \geq 0$). It is convenient to consider the extension $\tilde{\omega}_\beta$ (resp. $\tilde{\omega}_\infty$) of ω_β (resp. ω_∞) to the von Neumann algebra $\Pi_{\omega_\beta}(\mathscr{A})''$ (resp. $\Pi_{\omega_\infty}(\mathscr{A})''$). We write $\tilde{\omega}_\infty = (\Omega, \Omega)$.

Proposition 1. (Hugenholtz (1972) theorem 4.12 or Bratelli and Robinson (1981)): For $\beta < \infty$, $\tilde{\omega}_\beta$ is faithful and satisfies the KMS-condition with resp. to the extension $\tilde{\alpha}_t^\beta$ of α_t^β to $\pi_{\omega_\beta}(\mathscr{A})''$.

Proposition 2. $\tilde{\omega}_\infty$ is not faithful on $\pi_{\omega_\infty}(\mathscr{A}) \subset \pi\omega_\infty(\mathscr{A})''$.

Proof. We present two strategies of proving results like these. (Note that this statement is non-trivial for $\pi(\mathscr{A})$ in contrast to $\pi(\mathscr{A})''$. In fact, $\pi(\mathscr{A})$ is much smaller than $\pi(\mathscr{A})''$ which is the whole $B(\mathscr{H})$ in the irreducible case.)

(i) (*Irreducible case*). By the Kadison transitivity theorem (Dixmier 1969, p 44), to any vector $\psi \in \mathscr{H}_\omega$ in an irreducible representation π_ω of $\mathscr{A}$ there exists an element $A \in \mathscr{A}$ such that $\psi = \pi(A)\Omega$, furthermore one can construct a *Hermitian* element $B \in \mathscr{A}$ with $\pi(B)\psi = \psi$, $\pi(B)\Omega = 0$. This implies $\pi(BA)^*\Omega = \pi(A^*B)\Omega = 0$, $\pi(BA)\Omega = \psi \neq 0$.

(ii) (*General ground state representation*). We will exploit the notion of Arveson spectrum (cf e.g. Narnhofer *et al* 1983 and references given there) and choose $f \in L^1(\mathbb{R})$ such that $\tilde{f}(\lambda)$ has λ-support in $(0, \infty)$.

We then define $A_f := \int f(t)\alpha_t(A)\,\mathrm{d}t$ which lies again in $\mathscr{A}$. A_f has now its λ support (i.e. its Arveson spectrum, in more physical terms its energy support) contained in $(0, \infty)$. With $H_\omega \geq 0$ one can for every $\lambda \in \mathrm{spec.}(H_\omega)$ find an $A \in \mathscr{A}$ (by appropriately localising the λ supp with the help of a suitable $\tilde{f}$) such that λ-supp(A) is contained in an arbitrarily small neighbourhood of some $\lambda > 0$ and $\pi(A)\Omega \neq 0$ (cf Kastler 1976 § IV). On the other hand, A^* has its λ-supp contained in $(-\infty, 0)$ which implies $\pi(A^*)\Omega = 0$, namely again $\pi(A^*) \neq 0$ but $\pi(A^*)\Omega = 0$!

Remark. The second version of the proof is similar in spirit to a lemma proved in (Sirugue and Testard 1971, lemma 2.6). We thank one of the referees for this observation.

Propositions 1 and 2 display the basic differences between ground states and temperature states in statistical mechanics: for $\beta < \infty$, $\pi_\beta(\mathscr{A})''$ contains no annihilators of the state, while already $\pi_\infty(\mathscr{A})$ does. These differences are most clearly seen in the context of SSB by comparing Goldstone's theorem with its counterpart, a theorem of Coleman. This theorem, sometimes described by the sentence 'invariance of the vacuum is invariance of the world', may be informally stated in RQFT as follows.

Theorem. Let Q be the charge corresponding to a translation covariant local current. Then, if $Q\Omega=0$, then $\partial_\mu j^\mu(x)=0$ and therefore $[Q, H]=0$. A mathematical proof of a generalised and precisely formulated form of the above theorem was provided in (Gal-Ezer and Reeh 1974, 1975) to which we refer for references to the original papers.

This theorem contains essentially two assertions: $Q\Omega=0\rightarrow\partial_\mu j^\mu(x)=0\Rightarrow[Q, H]=0$. In non-relativistic theories $Q\Omega=0$ does not in general imply $\partial_\mu j^\mu(x)=0$, because of the eventual non-local character of the Lagrangian density. Hence, in statistical mechanics the latter should state that if the equilibrium is invariant under a continuous symmetry group with generator Q_β then $[Q_\beta, H_\beta]=0$ (in the sense of spectral projections). We summarise the (mostly known) results below.

Goldstone theorem	*Colemans theorem*
Equilibrium state $\tilde{\omega}$ not invariant under continuous symmetry group $\tilde{\gamma}_s$ but nevertheless the *formal* generator of the symmetry group commutes with the generator of time translations.	Equilibrium state $\tilde{\omega}$ invariant under continuous symmetry group $\tilde{\gamma}_s$.
$\beta<\infty$: H_β has no gap *not true* (but for reasons not directly† related to SSB, cf Haag *et al* 1974, prop. 3), the Goldstone phenomenon shows up however in a certain weak clustering (cf Landau *et al* 1981, Martin 1982) and a *characteristic singularity* at the *origin* in the joint $(H_\beta-P_\beta)$-spectrum (cf Requardt 1980)!	$[Q_\beta, H_\beta]=0$ *true* (see below).
$\beta=\infty$: H_∞ has no gap *true* (see Landau *et al* 1981 and references given therein).	$[Q_\infty, H_\infty]=0$ *not true* (see below).

Proof of Coleman's theorem for $\beta<\infty$: by a theorem of Takesaki (Hugenholtz (1972) theorem 7.2). $\tilde{\omega}_\beta$ defines uniquely a one-parameter automorphism group $\sigma_t^{\omega_\beta}$ of $\pi_{\omega_\beta}(\mathscr{A})''$ (the modular automorphism) and satisfies the KMS condition with respect to this automorphism group. Hence, by a theorem of Herman and Takesai in the context of von Neumann algebras (Herman and Takesaki 1970), see also (Sirugue and Takesaki 1970) or theorem 7.3 of Hugenholtz 1972) and the fact that $\tilde{\omega}_\beta$ is invariant under $\tilde{\gamma}_t^\beta$:

$$\sigma_t^{\omega_\beta}=(\tilde{\gamma}_t^\beta)^{-1}\sigma_t^{\omega_\beta}\tilde{\gamma}_t^\beta$$

which implies that $[Q_\beta, H_\beta]=0$ in the sense of spectral projections.

† There is, however, a symmetry being always spontaneously broken in systems with non-vanishing density, namely the Galilei boosts (Swieca 1967) and unpublished material of one of the authors (M Requardt). The Goldstone particles are, the *hydrodynamical phonons* (short-range interaction!).

Proof of theorem for $\beta=\infty$. Consider the infinite ferromagnetic Heisenberg model (Streater 1967) given by (for each finite $\Lambda \subset \mathbb{Z}^\nu$: for the definition in the infinite tensor product space, see Streater 1967):

$$H_\Lambda = -\sum_{i,j\in\Lambda} (J_{ij}^x S_i^x S_j^x + J_{ij}^y S_i^y S_j^y + J_{ij}^z S_i^z S_j^z)$$

$J_{ij}^z \geq 0$, $(|J_{ij}^x|, |J_{ij}^x|) \leq J_{ij}^z$, $J_{ij}^x \neq J_{ij}^y$ for some i, j. Then the infinite volume ground state still has 'all spins down' (that is, in the corresponding IDPS (Streater 1967), the properly defined infinite volume Hamiltonian is positive (Hepp 1972), and is therefore invariant under rotations around the z axis, with self-adjoint (unbounded) generator

$$Q_\infty = \frac{1}{2} \sum_{i\in\mathbb{Z}^\nu} (\sigma_i^z + \mathbb{1}).$$

However, $[Q_\infty, H_\infty] \neq 0$ if $J_{ij}^x \neq J_{ij}^y$ for some i, j.

The ground state of statistical mechanics is therefore distinct from the field theory ground state, where Coleman's theorem holds. This is true because the axioms of quantum field theory (including microcausality, which is not valid in statistical mechanics) imply that the vacuum is already a cyclic and separating vector for the von Neumann algebra of a spacetime region $R(\mathcal{O})$ (theorem 4.3 of Streater and Wightman 1964).

Although propositions 1 and 2 explain mathematically the differences between ground states and temperature states in the context of spontaneous symmetry breakdown in statistical mechanics, a physically more satisfactory explanation may be found from the point of view of stability under local perturbations of the dynamics (Bratelli 1978): if ω is a factor state on a L^1-asymptotically Abelian C^*-dynamical system $(\mathscr{A}, \tau)$ (Haag *et al* 1975, Bratelli *et al* 1978) satisfying the stability condition

$$\lim_{T\to\infty} \int_{-T}^{T} \mathrm{d}t\, \omega([A, \tau_t(B)]) = 0 \qquad (*)$$

$\forall A, B \in \mathscr{A}$, then ω is a extremal (τ, β)-KMS state for some $\beta \in \mathbb{R} U\{\pm\infty\}$ (theorem 6 of Bratelli *et al* 1978). However, a temperature state satisfying (*) is just a KMS state, but a ground state satisfying (*) not only satisfies the KMS ground state condition (Sirugue and Testard 1971)

$$H_\omega \geq 0$$

but also has a gap

$$H_\omega \geq \varepsilon > 0$$

(theorem 3 of Bratelli *et al* 1978). This is related to the fact that for a ground state the perturbation may cause the formation of an infinite number of infraparticles (Bratelli *et al* 1978).

Concerning properties of the generator of the symmetry, in RQFT we have the following important result (Maison 1972, see also Swieca 1970):

$$\lim_{R\to\infty} (\Omega|[Q_R, A]\Omega) = 2 \lim_{R\to\infty} (\Omega|Q_R \Delta\Omega)$$

for all $A \in \pi(\mathscr{A})$ (in the following we identify, for reasons of notational simplicity, A, B with $\pi(A), \pi(B)$).

This result can e.g. be used to prove the following physically desirable property†:

$$\lim_{R\to\infty}(A\Omega|Q_R B\Omega)=(A\Omega|QB\Omega)$$

in case the symmetry is conserved, i.e. Q being a well defined global generator, that is, we have

$$\lim_{R\to\infty}(A\Omega|Q_R B\Omega)=\lim_{R\to\infty}\{(A\Omega|[Q_R,B]\Omega)-(\Omega|A^*BQ_R\Omega)\}$$

$$=(A\Omega|QB\Omega)+\tfrac{1}{2}\underbrace{\lim_{R\to\infty}(\Omega|[Q_R,A^*B]\Omega)}_{=0!}=(A\Omega|QB\Omega).$$

One can ask the natural question to what extent this result holds in the non-relativistic regime. Somewhat surprisingly, the answer seems to depend on whether another (discrete) symmetry is conserved or not (compare Requardt 1982, last section).

Denoting time reflection by θ, it is implemented by an antiunitary operator if it is conserved. The physically relevant observables usually display a simple behaviour under θ, i.e. $\theta(A)=\varepsilon_A\cdot A$ with $\varepsilon_A=\pm 1$. Then we have the following proposition.

Proposition 3. Assuming that the symmetry γ_g is spontaneously broken with symmetry breaking observable A, local generators Q_R (cf the introductory remarks of this paper), if θ is conserved, i.e. $\theta\Omega=\Omega$, and if Q_R, A transform under θ as described above, we have:

$$(\Omega|[Q_R A]\Omega)=2(\Omega|Q_R A\Omega)$$

for R sufficiently large.

Proof.

$$(\Omega|AQ_R\Omega)=(\theta\Omega|AQ_R\theta\Omega)=\overline{(\Omega|\theta A\theta\theta Q_R\theta\Omega)}=\varepsilon_A\varepsilon_{Q_R}(AQ_R\Omega|\Omega)=\varepsilon_A\varepsilon_{Q_R}(\Omega|Q_R A\Omega)$$

(without loss of generality A can be chosen self-adjoint!). This yields

$$(\Omega|[Q_R,A]\Omega)=(1-\varepsilon_{Q_R}\varepsilon_A)(\Omega|Q_R A\Omega).$$

The LHS is *different from zero* for sufficiently large R since the symmetry is spontaneously broken, with A breaking the symmetry i.e. we have necessarily $\varepsilon_A=-\varepsilon_{Q_R}$. This implies

$$(\Omega|[Q_R,A]\Omega)=2(\Omega|Q_R A\Omega).$$

We see from this that the corresponding relation holds in the non-relativistic regime for a *symmetry breaking observable* and provided that *time reflection is conserved.* However in contrast to RQFT it does not hold in general, i.e. for all $A\in\pi(\mathscr{A})$, in particular nothing can be concluded for A's with $\varepsilon_A=\varepsilon_{Q_R}$.

In any case two interesting features can be inferred from the above relation. First, $\varepsilon_A=-\varepsilon_q$ for a symmetry breaking observable. Second, the range of $(\Omega|q(x,0)A\Omega)$ appears to be directly related to the *conservation* of time reflection! Note that in case of a *local* $q(x,0)$, A strictly local, (in fact the situation most frequently encountered), the LHS becomes independent of R‡ already for finite R. This implies immediately that

† This property implies in particular the hermiticity of the charge operator between states of type $A\Omega$ (see Swieca 1970).

‡ In fact, $(\Omega|[q(x,0),A]\Omega]\equiv 0$ for $|x|>R_A$! This implies $(\Omega|Q_R,A]\Omega)\equiv(\Omega|[Q_{R'},A]\Omega)$ for $R, R'>R_A$.

$(\Omega|q(x,0)A\Omega)$ has only a *finite* x-range (which is highly non-trivial!). In case of a not strictly local density q, e.g. the Hamiltonian density, the LHS acquires nevertheless a *finite* value in the limit (for short range interaction) so that also in this case a certain fall off of $(\Omega|q(x,0)A\Omega)$ can be inferred. There are in fact systems with $(\Omega|q(x,0)A\Omega)$ displaying a long-range behaviour. In that case *time reflection* has necessarily to be spontaneously broken! (Note that this does not contradict the picture that SSB implies long range order. It is usually the autocorrelation $(\Omega/A(x)A\Omega)-\langle A\rangle^2$ which displays long range correlations.)

As a case in point that proposition 3 is not true for arbitrary A's, choose the free Bose gas below the critical point. For $\beta=\infty$ the field operators have the form $\psi(x)=\psi_0(x)+\rho^{1/2}\,e^{i\varphi}$ with ψ_0 a Fock field, ρ, φ condensate density resp. phase. The local generators of the gauge symmetry have the form:

$$Q_R=j(f_R), \qquad j(x)=\psi^\dagger(x)\Psi(x)-(\Omega|\psi^\dagger(x)\Psi(x)\Omega).$$

As observable A we take $j(f)$, $\tilde{f}(0)\neq 0$. Then we have

$$\lim_{R\to\infty}(\Omega|[j(f_R),j(f)]\Omega)=0, \qquad \lim_{R\to\infty}(\Omega|j(f_R)j(f)\Omega)=\tilde{f}(0).$$

In the case $\beta<\infty$ we have the representation

$$\psi(f)=\Psi_0(\{1+\rho\}^{1/2}f)\otimes\mathbb{1}+\mathbb{1}\otimes\Psi_0^\dagger(\rho^{1/2}f)$$
$$\bar{\psi}(f)=\Psi_0^\dagger(\rho^{1/2}f)\otimes\mathbb{1}+\mathbb{1}\otimes\Psi_0(\{1+\rho\}^{1/2}f)$$

with $\bar{\psi}$ the representation in $\mathscr{A}'$. With $Q_R:=j(f_R)-j'(f_R)$, $j(f_R)\in\mathscr{A}$, $j'(f_R)\in\mathscr{A}'$ (more precisely, affiliated with $\mathscr{A}$ and $\mathscr{A}'$),

$$\lim_{R\to\infty}(\Omega|[Q_R,j(f)]\Omega)=0, \qquad \lim_{R\to\infty}(\Omega|Q_Rj(f)\Omega)=c\tilde{f}(0)$$

with $c\neq 0$ for a non-vanishing condensate density.

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